REAERATION EQUATIONS DERIVED FROM U.S. GEOLOGICAL SURVEY DATABASE

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ABSTRACT: Accurate estimation of the reaeration-rate coefficient (K_2) is extremely important for waste-load allocation. Currently, available K_2 estimation equations generally yield poor estimates when applied to stream conditions different from those for which the equations were derived because they were derived from small databases composed of potentially highly inaccurate measurements. A large data set of K_2 measurements made with tracer-gas methods was compiled from U.S. Geological Survey studies. This compilation included 493 reaches on 166 streams in 23 states. Careful screening to detect and eliminate erroneous measurements reduced the data set to 371 measurements. These measurements were divided into four subgroups on the basis of flow regime (channel control or pool and riffle) and stream scale (discharge greater than or less than 0.556 m^3/s). Multiple linear regression in logarithms was applied to relate K_2 to 12 stream hydraulic and water-quality characteristics. The resulting best-estimation equations had the form of semiempirical equations that included the rate of energy dissipation and discharge or depth and width as variables. For equation verification, a data set of K_2 measurements made with tracer-gas procedures by other agencies was compiled from the literature. This compilation included 127 reaches on at least 24 streams in at least seven states. The standard error of estimate obtained when applying the developed equations to the U.S. Geological Survey data set ranged from 44 to 61%, whereas the standard error of estimate was 78% when applied to the verification data set.

INTRODUCTION

The concentration of dissolved oxygen (DO) in natural waters is a primary indicator of the overall water quality and the viability of the aquatic habitat. Concentrations of DO in flowing streams are reduced as a result of biodegradation of carbonaceous and nitrogenous wastes discharged into the streams or deposited in the streambed sediment. Reaeration is the physical absorption of oxygen from the atmosphere by water. It is the most important natural means by which streams affected by waste inputs may recover DO. Further, the reaeration-rate coefficient (K_2) typically is the dominant parameter affecting the reliability of the simulation of DO concentrations in streams (Brown and Barnwell 1987, p. 175; Melching and Yoon 1996).

Waste-load allocation is the process by which allowable concentrations of constituents in discharge (allowable discharges) from wastewater-treatment plants (WWTPs) are determined such that acceptable water quality can be maintained in the stream. In the United States, minimum DO concentrations must be maintained for specified low-flow conditions (i.e., the 7-day, 10-year low flow). The allowable discharges from WWTPs typically are determined by simulation of water-quality processes in streams utilizing a computer model such as QUAL2E (Brown and Barnwell 1987). The waste-load allocation will affect multimillion-dollar decisions regarding the possible upgrading of WWTPs. These decisions require the most reliable data and simulations possible. Therefore, the K_2 value utilized in the simulation model must be determined carefully.

The purpose of this paper is to present a brief review of the current state of practice for determination of K_2 and the problems with the previously available equations for estimating K_2 and a detailed description of the development and testing of a new set of equations for estimating K_2. A set of new equations was developed on the basis of a large data set of 493 K_2 values on 166 streams in 23 states measured by the U.S. Geological Survey (USGS) utilizing gas-tracer methods. The set of new equations was verified utilizing 127 K_2 values on at least 24 streams in at least seven states measured by other agencies utilizing gas-tracer methods. Precise numbers of states and streams cannot be given because one source of verification data does not contain complete information on the source of the data.

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ESTIMATION OF REAERATION-RATE COEFFICIENT IN STANDARD PRACTICE

The value of K_2 can be measured accurately utilizing tracer-gas methods and field measurement of K_2 is strongly encouraged for reliable waste-load allocation. However, extensive field measurements of K_2 rarely are done for waste-load-allocation studies. Typically, K_2 values are determined using one of three approaches for waste-load-allocation studies. In the first approach, the concentrations of all the key constituents in the stream system are measured for a representative low-flow period, a simulation model is calibrated for this period, and the K_2-estimation equation from the literature that results in the best fit is used [e.g., New Jersey Department of Environmental Protection (1987)]. In the second approach, a limited number of K_2 values are measured for the stream system utilizing the tracer-gas method, and the best K_2-estimation equation from the literature for this stream system is determined on the basis of these measurements [e.g., Schmidt and Stamer (1987)]. In the third approach, a set of K_2 measurements is made for a group of streams representative of a region or state, and these measurements are used to derive K_2-estimation equations specific to that region or state, e.g., Cleveland (1989) for Texas and Hren (1984) for Ohio.

Generally, equations for estimation of K_2 based on stream hydraulic conditions are needed and applied because either K_2 measurements are not made or the low-flow conditions for which measurements are available may be substantially different from the low-flow conditions that must be simulated for waste-load allocation. These equations provide a rational means to estimate K_2 for the unmeasured low flows. Furthermore, in Europe water-quality management often is done on the basis of simulation of constituent concentrations throughout the entire year (e.g., Demuynck et al. 1997). Thus, equations are needed to relate K_2 to changing streamflow conditions throughout the year. More than 20 equations are available in the literature for estimation of K_2 on the basis of stream hydraulic characteristics. These equations and the data used to derive them are summarized in Flores (1998). Efforts in the literature to compare the various equations have determined the following problems with the equations:

1. Most of the K_2-estimation equations in the literature were derived from relatively small sets of laboratory or field data for a relatively localized group of streams. Wilson and Macleod (1974) applied 16 K_2-estimation equations (eight empirical equations utilizing flow velocity and depth and eight equations including an energy-dissipation term) to estimate K_2 values for a large number of field and laboratory measurements (482 measurements for the empirical equations and 382 measurements for the energy-dissipation equations). They found that each equation yielded accurate estimates for the data for which the equation was developed originally and yielded relatively poor estimates for almost all other data.
2. Most of the K_2-estimation equations in the literature developed using field data were derived from K_2 measurements obtained by the DO-balance or disturbed-equilibrium methods. Considering the errors in measuring the various components of the DO-balance and disturbed-equilibrium methods, Bennett and Rathbun (1972) estimated that the expected relative standard error of these methods are 65 and 115%, respectively. Thus, the data on which these equations are based include potentially substantial errors. Gas-tracer methods have been reported to have accuracies on the order of 10–25% (Tsivoglou et al. 1968; Rathbun and Grant 1978; Grant and Skavroneck 1980; Melching 1998). However, relatively few of the equations in the literature were derived from gas-tracer data [e.g., Tsivoglou and Wallace (1972), Hren (1984), Parker and Gay (1987), Cleveland (1989), and Parker and DeSimone (1992)].
3. The K_2 values measured in the laboratory are accurate, but it is uncertain how well laboratory conditions reflect reaeration in the field.

The USGS has done a large number of K_2 measurements utilizing tracer-gas methods in streams throughout the United States in cooperative projects with state and local agencies for the purposes of reaeration method development, waste-load allocation, and general characterization of water quality in streams. This database was analyzed to determine if more broadly applicable K_2-estimation equations could be developed on the basis of the large number of accurate K_2 measurements available. The results of this analysis are presented in the following sections.

DATA AVAILABLE

Prior to October 1996, the USGS had completed nearly 50 studies in cooperation with state, county, city, and regional agencies that involved the instream measurement of K_2-utilizing gas-tracer methods. The results of these studies have been reported in 41 USGS reports and papers and several sets of unpublished data. These reports and papers are listed in Flores (1998) and are not repeated here. In these studies, K_2 values have been measured for a total of 493 independent reaches on 166 streams in 23 states. The term independent reaches refers to reaches that are either distinctly different in space along the stream or multiple measurements at the same locations but for different flow conditons. The distribution of the measurements by state is listed in Table 1. For many reaches, several different tracer-gas methods of measuring K_2 were applied [e.g., Hren (1984) used both propane and ethylene]. The K_2 values used in this paper were the average values obtained from the various methods.

TABLE 1. National Distribution of Stream Reaches and Streams for which K_2 Measurements were Made by USGS

State (1)	Number of reaches (2)	Number of streams (3)
Alabama	16	8
Arizona	1	1
Arkansas	10	4
Colorado	68	10
Florida	34	12
Illinois	33	5
Indiana	6	2
Kentucky	10	6
Massachusetts	61	25
Mississippi	6	1
Missouri	18	5
Nevada	11	1
New Mexico	4	1
New York	26	14
North Dakota	12	2
Ohio	54	27
Oregon	13	1
South Carolina	7	3
Tennessee	24	5
Texas	6	1
Utah	9	1
Wisconsin	61	30
Wyoming	3	1

In addition to compiling the K_2 values for the various stream reaches, pertinent stream hydraulic and water-quality (where available) data were compiled. The hydraulic data compiled include the following:

1. Water-surface slope (S) (m/m). In several USGS studies, the value of S for the reach was measured directly by differential leveling or using gauge heights at fixed stations to determine the difference in water-surface elevation along the reach and the length of the reach. For other studies and reaches, steady uniform flow was assumed and the water-surface slope was set equal to the bed slope for the reach. The values of S included in the database range from 0.00001 to 0.06 m/m.
2. Discharge (Q) (m^3/s). The value of Q for a reach typically was determined as the average value from current meter measurements at the upstream and downstream ends of the reach. In cases in which the flow in the stream was considered uniform, values of Q measured for nearby cross sections or determined from stream gauges were utilized for the reach-average discharge. The values of Q included in the database range from 0.0028 to 210 m^3/s.
3. Mean streamflow velocity (V) (m/s). The value of V for a reach was determined from the traveltime measurements made in conjunction with the K_2 measurement and the length of the reach. In most cases, the centroidal traveltime for the dye curve was used. However, in some cases, centroidal traveltime data were not available and the peak traveltime was used. Examination of cases in which both centroidal and peak traveltime data were available indicated that the difference between these typically is small. The values of V included in the database range from 0.003 to 1.83 m/s.
4. Top width (W) (m). The value of W was determined as the average of top width values for the reach available from discharge-measurement notes and stream-geometry measurements. The values of W included in the database range from 0.78 to 162 m.
5. Depth (D) (m). The value of D was calculated from the continuity relation as $D = Q/(VW)$. This approach results

in a more representative value of the reach-averaged depth than the average of the depths at measurement locations. The values of D included in the database range from 0.0457 to 3.05 m.

Information on the bed material and flow regime (channel control or pool and riffle) also was compiled to describe stream hydraulics. Channel control refers to prismatic streams with relatively uniform flow properties. Water-chemistry data, including specific conductance, total solids concentrations, and methylene blue-active-substances concentrations (a measure of the presence of surfactants), also were compiled where available. Some of the studies reported data on wind velocity and direction, but the sources of the data were varied and not usable. A summary of the data utilized in the calibration of the K_2-estimation equations reported here is included in Flores (1998), and the data are available from the first author on request.

DATA SCREENING

Kilpatrick et al. (1989) note that the determination of the gas-transfer coefficient K_T (and subsequently K_2) may be subject to increasing error as the ratio of the upstream to downstream gas concentrations drops below 2.72 or, conversely, as $K_T T_t$ drops below 1, where T_t is the traveltime for the reach. In the planning of K_2-measurement studies, Kilpatrick et al. (1989) recommended that $K_T T_t \geq 1$. However, very few of the available field data meet this recommendation because of limitations on access to the stream, insufficient reach length caused by tributary streams, and other practical considerations. A low value of $K_T T_t$ indicates that the gas desorption time between sampling sections on a reach is insufficient for accurate measurement of the amount of desorption in the reach, and so the computed value of K_2 may be inaccurate. For the purpose of data screening for potential measurement outliers in this study, all measurements for which $K_T T_t \leq 0.3$ were omitted from the analysis. *If* it is assumed that the total error in measuring the gas and dye concentrations is 10%, *then* this data-screening criterion eliminates all K_T measurements with possible errors ≥33.3% (10%/0.3). Use of this criterion resulted in the omission of 89 K_2 measurements from the analysis. An additional 16 K_2 measurements were identified as outliers as a result of measurement problems or unusual stream conditions and were omitted from the analysis as described in Flores (1998). Finally, for 17 K_2 measurements, complete stream hydraulic data were not available and, thus, those measurements were omitted from the analysis.

DEVELOPMENT OF ESTIMATION EQUATIONS

Equational Forms

The equations previously developed for estimation of K_2 were obtained either by (1) multiple linear regression of the logarithms of the available data or (2) derivation of an equational form from a theoretical concept of the reaeration process. Some of the coefficients and exponents of the theoretical equations were determined from physical reasoning, whereas others were determined through a least-squares fitting of available data. Thus, the equations determined from multiple linear regression are categorized as empirical models, and the equations determined by least-squares fitting of a theoretically derived equation are categorized as semiempirical models (Rathbun 1977).

In the analysis of the data to determine appropriate equations for estimation of K_2 on the basis of stream hydraulic and water-quality characteristics, two approaches were taken to be consistent with previous estimation equation development. The first approach was application of multiple linear regression to the logarithms of the data resulting in an empirical model. The second approach was determination of a least-squares fit of several semiempirical models proposed in the literature. These semiempirical models were evaluated because the theoretical concepts utilized to develop these models might result in a more broadly applicable estimation equation than could be obtained from multiple linear regression. The multiple linear regression included the rate of energy dissipation as a possible independent variable. Thus, the semiempirical equations of the energy-dissipation form listed in Rathbun (1977) were considered in the multiple linear regression analysis.

The multiple linear regression in logarithms of base 10 resulted in estimation equations of the form

$$K_2 = aH_1^{b1}H_2^{b2} \ldots q_1^{c1}q_2^{c2} \ldots \tag{1}$$

where K_2 is in days^{-1} and is for a temperature of 20°C; H_i = stream hydraulic characteristics; q_j = water-quality characteristics; a = a coefficient; bi = exponents corresponding to stream hydraulic characteristics i; and cj = exponents corresponding to water-quality characteristics j. The water-quality characteristics considered included specific conductance, total solids concentrations, and/or methylene blue-active-substances concentrations. The stream hydraulic characteristics considered included S, Q, V, W, and D (discussed earlier) and combinations of these characteristics that have physical meaning. These combined characteristics included the following:

1. The rate of energy dissipation (m/s) as approximated by Tsivoglou and Wallace (1972) as

$$\Delta H/T_t = SL/(L/V) = VS \tag{2}$$

where ΔH = change in energy head over the reach; and L = length of the reach.

2. Manning's roughness coefficient n (dimensionless) computed as

$$n = R^{2/3}S^{1/2}/V \tag{3}$$

where R = hydraulic radius computed assuming a rectangular channel [i.e., $R = DW/(W + 2D)$].

3. The shear velocity u_* (m/s) computed as

$$u_* = (gRS)^{1/2} \tag{4}$$

where g = acceleration of gravity (m/s^2).

4. The Froude number F (dimensionless) computed as

$$\mathsf{F} = V/(gD)^{1/2} \tag{5}$$

The semiempirical model forms fit to the data included:

1. The Dobbins (1965) conceptual model based on a film-penetration model

$$K_2 = a\,\frac{1.0 + \mathsf{F}^2}{(b + \mathsf{F})^{1.5}}\,\frac{(VS)^{0.375}}{D}\coth\frac{c(VS)^{0.125}}{(b + \mathsf{F})^{0.5}} \tag{6}$$

2. The Lau (1972) semiempirical model based on a molecular-diffusion model

$$K_2 = a(u_*/V)^b(V/D) \tag{7}$$

3. The Parkhurst and Pomeroy (1972) semiempirical model based on an energy-dissipation model and a molecular-diffusion model

$$K_2 = a(1 + b\mathsf{F}^2)(VS)^cD^d \tag{8}$$

4. The Thackston and Krenkel (1969) semiempirical model based on a surface-renewal (turbulent-diffusion) model

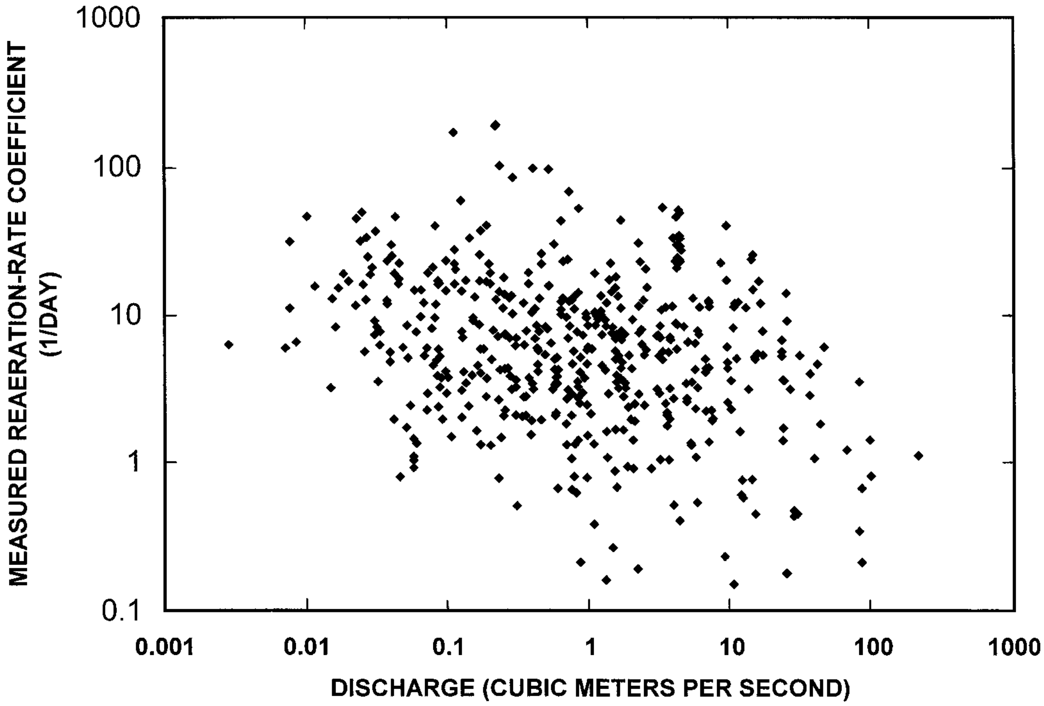

FIG. 1. Relation between Reaeration-Rate Coefficient K_2 and Discharge

$$K_2 = a(1 + \mathsf{F}^b)u_\cdot D^{-1} \quad (9)$$

In (6)–(9), a, b, c, and d represent the coefficients and exponents that were estimated by least-squares fitting. The values of the other coefficients and exponents in (6)–(9) were derived by the authors from physical reasoning and/or theoretical considerations and, thus, were not fitted to the data.

Fitted Equations

In the development of the estimation equations reported here, the edited data were subdivided on the basis of flow regime into two groups: pool and riffle streams and channel-control streams. Analysis and observation of the data in each of these groups indicated the need to further divide the groups by stream scale. No relation between discharge and K_2 is apparent from the data (Fig. 1). However, K_2 is known to increase with velocity and decrease with depth; both generally increase with discharge. Thus, as discharge increases the relative effects of the increase in velocity and the increase in depth on the value of K_2 may vary among streams. Nearly all the K_2 measurements were made for low-flow conditions on the respective streams. From analysis of the data, a discharge of 0.566 m^3/s provided a convenient value for division of the data based on stream scale reflecting a break in the relative importance of the increase in velocity and the increase in depth. Further, a discharge of 0.556 m^3/s was used by Tsivoglou and Neal (1976) in the definition of their energy-dissipation model of K_2 estimation.

Equations were fitted to the K_2 measurements for each of the four subgroups of the edited data utilizing multiple linear regression and least-squares fitting of semiempirical equational forms. For each subgroup, the estimation equation obtained from multiple linear regression yielded the best-estimation equation in terms of the standard deviation of the logarithms and the coefficient of variation. The following is the best-estimation equation for each subgroup:

1. Pool and riffle streams, low flow ($Q < 0.556$ m^3/s) derived from 99 K_2 measurements

$$K_2 = 517(VS)^{0.524}Q^{-0.242} \quad (10)$$

2. Pool and riffle streams, high flow ($Q > 0.556$ m^3/s) derived from 130 K_2 measurements

$$K_2 = 596(VS)^{0.528}Q^{-0.136} \quad (11)$$

3. Channel-control streams, low flow ($Q < 0.556$ m^3/s) derived from 77 K_2 measurements

$$K_2 = 88(VS)^{0.313}D^{-0.353} \quad (12)$$

4. Channel-control streams, high flow ($Q > 0.556$ m^3/s) derived from 65 K_2 measurements

$$K_2 = 142(VS)^{0.333}D^{-0.66}W^{-0.243} \quad (13)$$

A statistical summary of the quality of fit of these equations including the multiple correlation coefficient, the standard error of estimate of the logarithms, the coefficient of variation of the transformed equations, and the standard error of the estimated K_2 values in real space is given in Table 2. Scattergrams illustrating the overall fit quality of (10) and (11) for pool and riffle streams and (12) and (13) for channel-control streams are shown in Figs. 2 and 3, respectively.

The least-squares fits of the equational forms of Dobbins (1965), Thackston and Krenkel (1969), and Lau (1972) generally resulted in much poorer fits of the data than obtained by multiple linear regression with coefficients of variation 15–57% higher. However, in terms of the correlation coeffi-

TABLE 2. Fit Statistics for K_2-Estimation Equations Developed for Edited USGS Database Using Multiple Linear Regression

Equation (1)	Correlation coefficient (2)	Standard error of logarithms (3)	Coefficient of variation (4)	Standard deviation in real space (5)
Pool and riffle $Q < 0.556$ m^3/s	0.835	0.244	0.610	23.8
Pool and riffle $Q > 0.556$ m^3/s	0.900	0.183	0.441	5.36
Channel control $Q < 0.556$ m^3/s	0.690	0.238	0.591	6.94
Channel control $Q > 0.556$ m^3/s	0.845	0.241	0.601	7.56

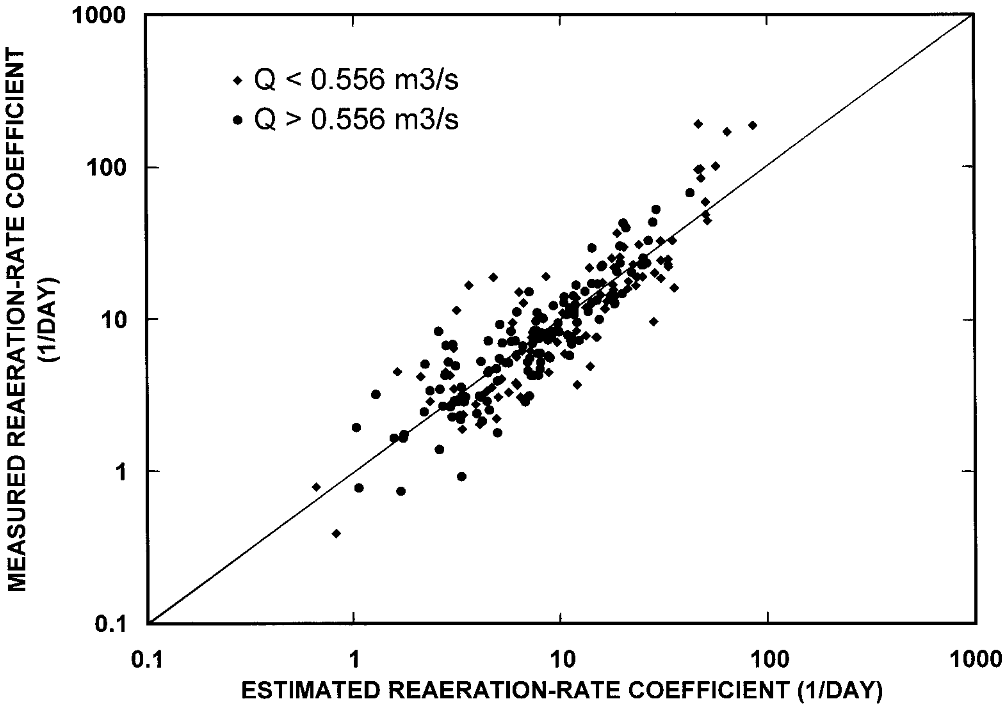

FIG. 2. Reaeration-Rate Coefficients K_2 Measured and Estimated with Equations Developed from Multiple Linear Regression for Pool and Riffle Streams in USGS Database

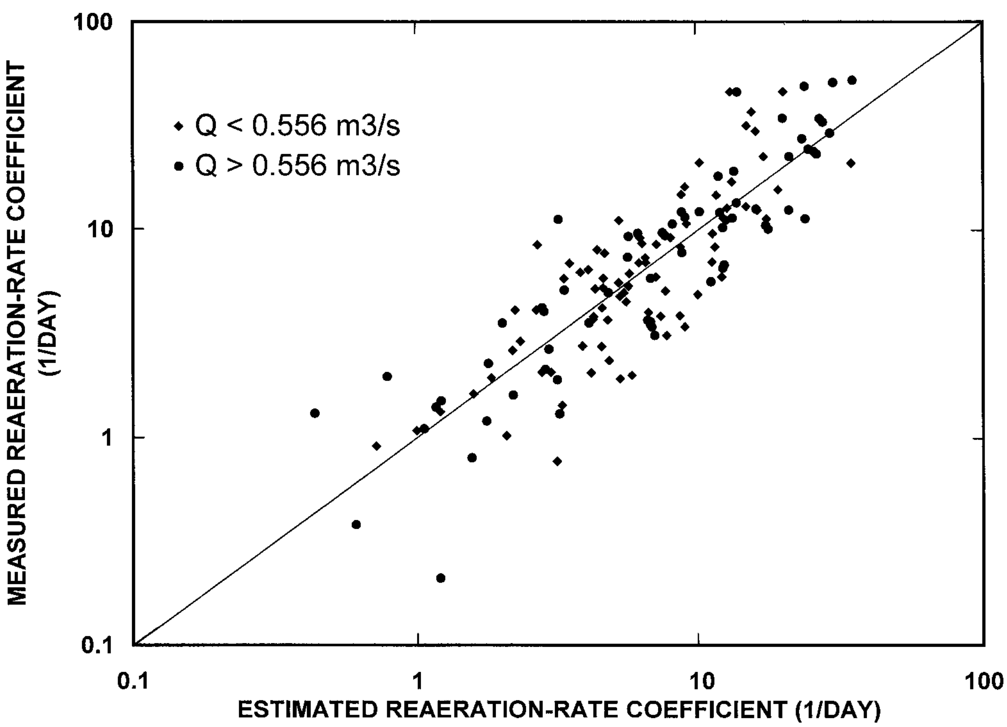

FIG. 3. Reaeration-Rate Coefficients K_2 Measured and Estimated with Equations Developed from Multiple Linear Regression for Channel-Control Streams in USGS Database

TABLE 3. Coefficients and Exponents for Modified Parkhurst and Pomeroy Equations

Equation (1)	*a* (2)	*b* (3)	*c* (4)	*d* (5)
Pool and riffle $Q < 0.556$ m^3/s	1,788	0.724	0.767	−0.135
Pool and riffle $Q > 0.556$ m^3/s	765	−1.016	0.661	−0.412
Channel control $Q < 0.556$ m^3/s	36.8	−0.569	0.179	−0.539
Channel control $Q > 0.556$ m^3/s	34.7	4.26	0.189	−0.421

cient (in 3 of 4 cases) and standard error of the estimated K_2 values in real space, the fitted Parkhurst and Pomeroy (1972) equation provided the best fit. The fitted coefficients and exponents for the modified Parkhurst and Pomeroy equation are listed in Table 3. The fitted coefficients are radically different from the values proposed by Parkhurst and Pomeroy ($a = 25.9$, $b = 0.17$, $c = 0.375$, and $d = -1$) and among the four data subgroups. Similar variations among the fitted coefficients and exponents were obtained for the modified Dobbins, Lau, and Thackston and Krenkel equations. These results indicate that the conceptual basis of these equational forms has been lost in the curve-fitting process and, thus, the resulting fitted coefficients for the equational forms are not suitable for K_2 estimation. Complete details on the fitted semiempirical equational forms are available in Flores (1998).

Because the variance of the differences between measured and estimated K_2 values in real space is not constant over the range of K_2, the standard deviation in real space is not a good measure of the estimation accuracy of the four equations. A constant variance for the differences results for the logarithmically transformed estimated and measured K_2 values. The coefficient of variation is the ratio of the standard deviation to the mean of the data. If the estimated value of K_2 obtained from the appropriate equation is considered as the expected value (mean value) of K_2 for the streamflow conditions, the coefficient of variation gives the standard error of estimate in fractional (percentage if multiplied by 100) terms. Thus, (10)–(13) constitute the overall best-estimation equations with standard errors of estimate ranging between 44 and 61%.

It is interesting that the empirical process of multiple linear regression resulted in best-fit estimation equations with a semiempirical, energy-dissipation form. Thus, based on a large data set for a wide variety of streamflow conditions, it appears that a strong relation exists between energy dissipation and K_2. Furthermore, from a conceptual viewpoint, the form of the fitted equations seems to indicate that the relation between the rate of energy dissipation and the reaeration rate coefficient is regulated by the stream scale. For channel-control conditions, the mean stream width and depth adequately describe stream scale, whereas for pool and riffle conditions, the mean stream width and depth are difficult to determine and, thus, discharge serves as a surrogate for stream scale.

Water-quality-characteristic data (specific conductance, total solids concentrations, and/or methylene blue-active-substances concentrations) were only available for a limited set of the K_2 measurements. However, for this limited data set, multiple linear regression indicated that none of the water-quality characteristics significantly affected the estimation of K_2.

Equation Verification

To verify the usefulness of the equations developed on the basis of the USGS database, a literature search was done to identify tracer-gas measurements of K_2 made by other agencies for which sufficient stream hydraulic data were available to test the K_2-estimation equations developed here. The search began with the expert system for K_2 determination developed by Whittemore (1990). Values of K_2 and a limited selection of stream hydraulic characteristics for more than 1,200 tracer-gas measurements of K_2 were compiled and entered into the expert system. Only a limited number of these measurements (1) included the stream hydraulic characteristic data needed to test the estimation equations (S, V, Q, W, and D depending on stream conditions), (2) were for independent reaches, and (3) were not collected by the USGS. Additional data were obtained from Tsivoglou and Wallace (1972), Shindala and Traux (1980), and Cleveland (1989).

The streamflow regime was identified for only aproximately 30 reaches of pool and riffle streams in the potential database for estimation-equation verification. The remainder of the streams in the verification database also were assumed to be pool and riffle because pool and riffle hydraulics tend to dominate the low-flow conditions of interest for waste-load allocation and K_2 measurement. With this assumption and the application of the data-screening criteria described earlier, 127 K_2 measurements on at least 24 streams in at least seven states (Colorado, Georgia, Kentucky, Michigan, Mississippi, North Carolina, and Texas) were available for equation verification. Precise numbers of states and streams cannot be given because the expert system does not contain complete information on the source of the data. A summary of the data utilized in the verification of the K_2-estimation equations reported here is included in Flores (1998).

The verification data are estimated with (10) and (11) nearly as well as the data used to derive these equations. Statistically, the overal fit statistics are a correlation coefficient of 0.87, a standard error of estimate of the logarithms of 0.32, a coefficient of variation of the transformed equations of 0.850, and a standard error of the estimated K_2 values in real space of 4.59. For 36 of the 127 reaches used for verification, (12) or (13) yielded a better estimate of K_2 than (10) or (11), respectively. Because the flow regime is unknown for most reaches, if the best estimate of K_2 from (10) or (12) or (11) or (13) is

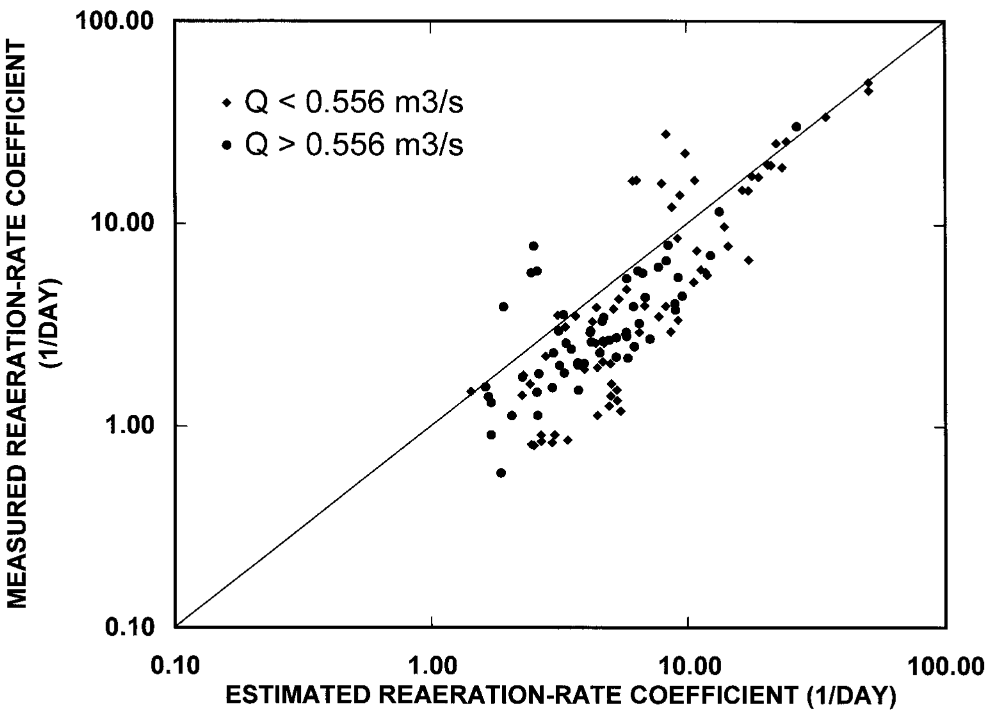

FIG. 4. Reaeration-Rate Coefficients K_2 Measured and Estimated with Equations Developed from Multiple Linear Regression for Pool and Riffle Streams in Verification Database

considered, the statistics are a correlation coefficient of 0.90, a standard error of estimate of the logarithms of 0.30, a coefficient of variation of the tranformed equations of 0.782, and a standard error of estimated K_2 values in real space of 3.84. A scattergram illustrating the overall fit quality of the best estimate from (10) and (11) or (12) and (13) for the verification data set is shown in Fig. 4.

CONCLUSIONS

A large data set of reaeration-rate coefficient (K_2) measurements made with tracer-gas methods was compiled from USGS cooperative studies. This compilation included 493 reaches on 166 streams in 23 states. After careful screening to detect and eliminate potentially erroneous measurements, the data set was reduced to 370 measurements. These measurements were divided into four subgroups on the basis of flow regime and stream scale (defined by discharge) as follows: pool and riffle streams with $Q > 0.556$ m^3/s and $Q < 0.556$ m^3/s and channel-control streams with $Q > 0.556$ m^3/s and $Q < 0.556$ m^3/s. Multiple linear regression in logarithms was applied to relate K_2 to 12 stream hydraulic and water-quality characteristics including water-surface slope, discharge, velocity, width, depth, rate of energy dissipation, Manning's n, shear velocity, Froude number, specific conductance, total solids, and methylene blue-active substances. Least-squares fitting also was applied to several semiempirical equational forms proposed in the literature to see if the theoretical basis of these equations could yield better estimation equations than obtained by regression.

The resulting best-estimation equations were obtained from multiple linear regression and had the form of semiempirical equations, including the rate of energy dissipation [(10)–(13)]. For equation verification, a data set of K_2 measurements made with tracer-gas measurements by other agencies was compiled from the literature. This compilation included 127 reaches on at least 24 streams in at least seven states. The equations derived as part of this study estimated K_2 values for the verification data set nearly as well as they estimated K_2 for the USGS data set. The standard error of estimate obtained when applying the equations developed here to the USGS database ranged from 44 to 61%, whereas the standard error of estimate was 78% when applied to the verification database.

The equations developed here have a semiempirical, energy-dissipation form and provide reliable estimates of K_2 for a wide range of streamflow conditions. Thus, these equations may be reliable for estimation of K_2 for waste-load-allocation studies for which instream measurements of K_2 are not made because of financial constraints. However, these equations are not a replacement for field measurement of K_2 on the streams of interest and in stream measurements of K_2 should be done whenever time and budget allow.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

- a = coefficient computed in multiple linear regression analysis or least-squares equation fitting;
- b, c, d = coefficients and exponents of estimation equations determined by least-squares fitting;
- bi = exponents corresponding to stream hydraulic characteristics i in multiple linear regression analysis;
- cj = exponents corresponding to water-quality characteristics j in multiple linear regression analysis;
- D = reach-average flow depth;
- F = Froude number;
- g = acceleration of gravity;
- ΔH = change in energy head over reach;
- H_i = stream hydraulic characteristics considered in multiple linear regression analysis;
- K_2 = reaeration-rate coefficient in days^{-1} for temperature of 20°C;
- K_T = gas desorption-rate coefficient;
- L = reach length;
- n = Manning's roughness coefficient;
- q_j = water-quality characteristics considered in multiple linear regression analysis;
- Q = reach-average discharge;
- R = reach-average hydraulic radius;
- S = water-surface slope over reach;
- T_t = peak or centroidal traveltime of dispersion/dilution tracer depending on reaeration-rate coefficient measurement method being used;
- u_* = shear velocity;
- V = reach-average velocity;
- VS = rate of energy dissipation over reach; and
- W = reach-average flow top width.