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Comparison of Estimates of Uncertainty of Discharge at U.S. Geological Survey Index-Velocity Gages on the Chicago Sanitary and Ship Canal, Illinois

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Abstract

Estimates of uncertainty of discharge at time scales from 5 minutes to 1 year were obtained for two index-velocity gages on the Chicago Sanitary and Ship Canal (CSSC), Ill., instrumented with acoustic velocity meters (AVMs). The velocity measurements obtained from the AVMs are corrected to a mean channel velocity by use of an index-velocity rating (IVR). The IVR is a regression-derived relation between the AVM velocity estimates and those obtained using acoustic Doppler current profilers (ADCPs). The uncertainty estimation method is based on the first-order variance method, but the AVM velocity error is estimated from an empirical perspective, using the statistics of the IVR regression. It is not clear whether to include the standard error of the IVR regression (σ_ε^2) in the discharge uncertainty. At the 5-minute time scale when σ_ε^2 is included, σ_ε^2 has the dominant contribution to the discharge uncertainty, and the discharge uncertainty (expressed as the standard deviation of the discharge estimate) is about 5 m^3/s at one gage and 8 m^3/s at the other, independent of discharge. When σ_ε^2 is not included, the discharge uncertainty at the 5-minute time scale is much smaller (about 0.5 m^3/s) and depends more strongly on discharge. For time scales 1 day or greater and when σ_ε^2 is not included, the uncertainty of the IVR parameters dominates the discharge uncertainty, and the value of the discharge uncertainty is about 0.4 m^3/s for one gage and 0.5 m^3/s for the other gage.

INTRODUCTION

The total uncertainty of discharge measurements traditionally has been estimated as the square root of the summation of the squares of the total uncertainties from different sources (e.g., Carter and Anderson, 1963; Simpson and Oltman, 1993). In the International Organization for Standardization (ISO) 6416 standard (ISO, 1992), the ISO recommends estimating the total uncertainty of measurements from acoustic velocity meters (AVMs) with a method based on the following approach. The total uncertainty of AVM measurements is estimated as the square root of the sums of the squares of the uncertainties of the contributing sources weighted with prescribed factors; however, the values of these factors are not clearly justified. More advanced methods for estimating total uncertainty gradually are becoming part of today's engineering practice. One such method is the first-order-variance method (Ang and Tang, 1984; Tung and Yen, 1992; Muste and Stern, 2000). The purpose of this paper is to describe the application of this method to the estimation of discharge uncertainty at gages on the CSSC (Figure 1).

The first-order variance method has two fundamental components: (1) the measured or estimated value of some quantity X is considered to be the sum of a fixed and true but unknown value X' and an independent, mean zero error term ε_X; that is, $X = X' + \varepsilon_X$; and (2) the estimation error of a quantity Y that is a function of one or more measured or estimated quantities arises because of the error of estimation of the variables from which it is computed, and is computed according to a first-order approximation to the complete variance formula. Formally, for a quantity $Y = g(X_1, X_2, \ldots, X_n)$, in the first-order variance method, the uncertainty of Y is the first-order approximation to the variance of Y,

$$\sigma_Y^2 \approx \sum_{i=1}^{n}\sum_{j=1}^{n} \left.\frac{\partial g}{\partial X_i}\right|_{\mu} \left.\frac{\partial g}{\partial X_j}\right|_{\mu} \sigma_{X_i,X_j} \,, \qquad (1)$$

(see, for example, Benjamin and Cornell (1970, p. 184)) where $\left.\frac{\partial g}{\partial X_i}\right|_{\mu}$ indicates the evaluation of the partial derivative at the mean μ, and σ_{X_i,X_j} indicates the covariance of X_i and X_j. As the mean μ (the true value) is not known, the partial derivatives are, in practice, evaluated at the observations, which have the same mean as the true values. Equation (1) may be derived from a multi-dimensional Taylor series expansion of $Y = g(X_1, X_2, \ldots, X_n)$. In the case that the variables $X_1, X_2, \ldots, X_n$ are mutually statistically independent, equation (1) reduces to the form in which the first-order variance method usually is expressed,

$$\sigma_Y^2 \approx \sum_{i=1}^{n} \left(\left.\frac{\partial g}{\partial x_i}\right|_{\mu} \right)^2 \sigma_{X_i}^2 \,. \qquad (2)$$

APPLICATIONS TO DISCHARGE MEASUREMENTS INVOLVING ACOUSTIC VELOCITY METERS AND INDEX-VELOCITY RATINGS

In the case of the discharge measurements considered here, the dependent variable is the discharge Q, and the independent variables are the velocity V and the cross-sectional area *A*. As separate instruments are used to obtain velocity and area, *V* and *A* usually can be taken to be independent. Therefore, the simpler formula (2) may be used, and the basis of the analysis here is the expression

$$\sigma_Q^2 \approx \left(\frac{\partial Q}{\partial V}\bigg|_\mu \right)^2 \sigma_V^2 + \left(\frac{\partial Q}{\partial A}\bigg|_\mu \right)^2 \sigma_A^2 = A^2\sigma_V^2 + V^2\sigma_A^2 . \qquad (3)$$

In order to evaluate equation (3), estimates of σ_A^2 and σ_V^2 clearly are needed. In the present case and throughout the U.S. Geological Survey streamgaging program when AVMs are used, *V* is estimated with a index-velocity relation obtained by regressing the line velocity V_L ; that is, the velocity obtained using the AVM(s) at the site, against a presumably less biased estimate of *V*. In this case, velocity measurements obtained using acoustic Doppler current profilers (ADCPs). For purposes of this paper, the ADCP estimates of *V* are assumed to be unbiased estimates of the true velocity. For discussion of ADCP measurement, see Simpson and Oltmann (1993) and Gordon (1989). This results in *V* and V_L being related as

$$V = \alpha + \beta V_L + \varepsilon , \qquad (4)$$

where α and β are obtained using ordinary least-squares (OLS) linear regression, and ε is a statistically independent, mean zero error term. In some cases, the intercept α may be taken to be zero and so terms involving α drop out. The velocity variance σ_V^2 is then given by

$$\sigma_V^2 = \sigma_\alpha^2 + V_L^2\sigma_\beta^2 + \sigma_\varepsilon^2 + 2V_L\sigma_{\alpha,\beta} . \qquad (5)$$

The quantities σ_α^2, σ_β^2, $\sigma_{\alpha,\beta}$, and σ_ε^2 are obtained from the regression statistics (for example, see Mood and Graybill (1963, p. 333)). The inclusion of σ_ε^2 in equation (5) is debatable in the following sense. The error ε accounts for both measurement error in *V* and model error; that is, variability in *V* because of factors that are not included in the prediction equation, which includes only line velocity. If the error in measuring *V* is the dominant component of ε, then the mean value of the prediction, $\alpha + \beta V_L$, is the correct value of *V* and the uncertainty arises only from the regression coefficients α and β. On the other hand, if the measurement error in *V* is small relative to ε, then the model error is significant, $\alpha + \beta V_L$ is not the correct value of *V*, and the additional uncertainty σ_ε^2 needs to be included. As the information available at this time does not resolve this debate (what is needed is an independent estimate of error of measurement of *V* by an ADCP), including or excluding σ_ε^2 in equation (5) provides upper and lower bounds, respectively, on the possible uncertainty in *V*.

The remaining unknown in equation (3) is the uncertainty in cross-sectional area, *A*. *A* usually is estimated by using a measured stage *h* into a stage-area rating curve, $A(h)$.

In the CSSC, because the channels have nearly vertical straight walls in the range of observed stages, the stage-area rating is well-described by the linear equation

$$A(h) = a + bh \,. \tag{6}$$

In general, the uncertainty in cross-sectional area would include the effects of uncertainty in the parameters describing the stage-area rating (*a* and *b*) because of surveying errors and variation in the cross-section over the channel reach covered by the acoustic path. However, in the present case, these errors are eliminated because the ADCP velocity used in the index-velocity relation originally is measured as a discharge and is converted to a velocity by dividing by the estimated *A*. The only remaining error is in the measurement of the stage, *h*, itself. The largest term of stage measurement error arises because of the recording of the stage to the nearest 0.003 m (0.01 ft). Therefore, applying the first-order variance approach to the expression for area (equation (6)) under the assumption that *a* and *b* are non-random gives

$$\sigma_A^2 = b^2 \sigma_h^2 \,. \tag{7}$$

Given this estimate of σ_A^2, the error of a single, quasi-instantaneous discharge measurement, σ_Q^2, can be computed using equations (3), (5), and (7).

Estimation of Error in Time-Averaged Discharge

In many cases, including the one described here, it is important to estimate the error in discharge not only at the unit value time scale, but also for discharge averaged over some time period, such as a day, month or year. The general methodology for error of time-averaged discharge is as follows. The variance temporally averaged discharge, $\overline{Q} = n^{-1}\sum_{t=1}^{n} Q_t$, is given by $\sigma_{\overline{Q}}^2 = n^{-2}\sigma_{\sum_{t=1}^{n} Q_t}^2$, where

$$\sigma_{\sum_{t=1}^{n} Q_t}^2 = \sum_{t=1}^{n} \sigma_{Q_t}^2 + 2\sum_{t=1}^{n-1}\sum_{s=t}^{n} \sigma_{Q_t, Q_s} \,. \tag{8}$$

The first term on the right-hand side of equation (8) is a simple sum of instantaneous discharge variances. The second term arises because of factors common to the errors in discharge at different times; these errors include the errors in the index-velocity relation parameters α and β, and auto-correlation in ε and in measurement errors of the stage *h*. The stage errors have a negligible effect on the discharge errors, so auto-correlation of stage errors is similarly unimportant. Whereas it was difficult to assess the degree of auto-correlation in ε from the present data, by testing different auto-correlation models it was determined that auto-correlation in ε does not contribute appreciably to the total error in the average discharge. Further simplification of the right-hand side of equation (8) is possible because stage and, thus, area do not vary appreciably in time at these sites, therefore $A(t) \approx \overline{A}$. Following these simplifications, the covariance of discharge is approximately

$$\sigma_{Q_t,Q_s} \approx \overline{A}^2\left[\sigma_\alpha^2 + \left(V_L(s)+V_L(t)\right)\sigma_{\alpha,\beta} + V_L(s)V_L(t)\sigma_\beta^2\right]. \tag{9}$$

Using equation (8) with equations (3), (5) and (7) and dividing by n^2 gives the variance of time-averaged flows, $\sigma_{\overline{Q}}^2$.

It further may be shown that even though the AVM velocity, V_L, does vary widely at the gages, for longer time periods such as a year, the computation of $\sigma_{\overline{Q}}^2$ is affected negligibly by assuming that $V_L(t)$ is equal to its time average denoted by $\overline{V}_L$. Equation (9) then may be written as

$$\sigma_{Q_t,Q_s} \approx \overline{A}^2\left[\sigma_\alpha^2 + 2\overline{V_L}\sigma_{\alpha,\beta} + \overline{V_L}^2\sigma_\beta^2\right]. \tag{10}$$

Making similar approximations in equations (3) and (5) gives σ_Q^2 as

$$\sigma_Q^2 \approx \left(\alpha + \beta\overline{V}_L\right)^2\sigma_A^2 + \overline{A}^2\left[\sigma_\alpha^2 + 2\overline{V_L}\sigma_{\alpha,\beta} + \overline{V_L}^2\sigma_\beta^2 + \sigma_\varepsilon^2\right]. \tag{11}$$

As there are *n* variance terms and $\left(n^2 - n\right)/2$ covariance terms, the variance and covariance terms in equation (8) may be added using equations (10) and (11) to obtain simplified estimates of $\sigma_{\overline{Q}}^2$ as

$$\sigma_{\overline{Q}}^2 \approx n^{-1}\left[\left(\alpha + \beta\overline{V}_L\right)^2\sigma_A^2 + \overline{A}^2\sigma_\varepsilon^2\right] + \overline{A}^2\left[\sigma_\alpha^2 + 2\overline{V_L}\sigma_{\alpha,\beta} + \overline{V_L}^2\sigma_\beta^2\right]. \tag{12}$$

From equation (12) it is clear that certain errors (the random errors arising from measurements of area and IVR regression error ε) decrease as n^{-1} with the number of time intervals *n* being averaged, whereas other errors (those arising from uncertainty in the index velocity relation) are independent of *n*. For these reasons, decreasing the error in average discharge over long time periods, such as a month or year, would require additional ADCP measurements to reduce the uncertainty in the IVR.

APPLICATION TO DISCHARGE MEASUREMENTS AT TWO GAGES ON THE CHICAGO SHIP AND SANITARY CANAL

In addition to water-supply withdrawals from Lake Michigan, the State of Illinois diverts water from Lake Michigan directly into the CSSC at three locations: Wilmette, the Chicago River Controlling Works (CRCW) near downtown Chicago, and O'Brien Lock and Dam (see Figure 1). A series of U.S. Supreme Court decrees limits the amount of water that the State is allowed to divert (Wisconsin v. Illinois, 1930, 1933, 1967, 1980). The U.S. Geological Survey (USGS) has been responsible for measuring diversion at a gage on the Chicago Sanitary and Ship Canal (CSSC) at Romeoville, Illinois, since October 1984. Flow at this gage is controlled by the operation of turbines and the lock about 8 km downstream at Lockport near the downstream end of the CSSC near where it discharges into the Des Plaines River. In 1996, the USGS was asked by the Illinois Department of Natural Resources to install three additional gages at the lakefront diversion sites in order to measure the direct diversion. The Chicago River at Columbus Dr. gage is one of these. It is located about 0.5 km inland (west) of the diversion point at the CRCW. When the CRCW are closed, flow at the Columbus Dr. station is low (5 m^3/s or less) and sometimes bi-directional, especially during winter. During summer storms, flow may reverse and discharge into Lake Michigan.

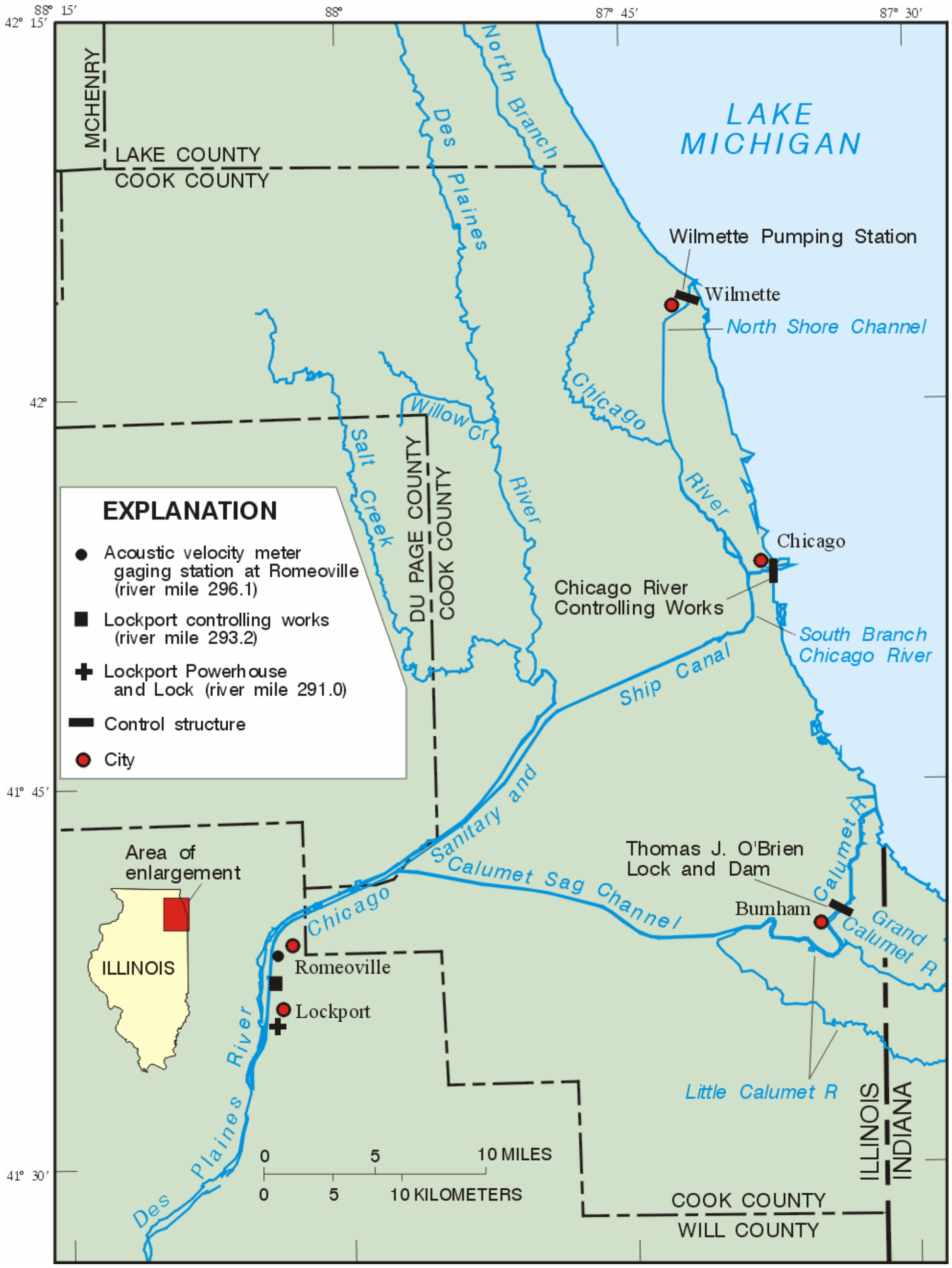

Figure 1. Location of U.S. Geological Survey AVM gages on the Chicago Ship and Sanitary Canal, Illinois.

AVMs are utilized in all four of the CSSC gages to measure flow. AVMs are required because of the complex site hydraulics characterized by unsteady flow conditions, backwater, and low velocities. AVMs transmit sound waves across the channel at a known angle to the flow direction. The difference between the upstream and downstream travel times for the sound wave is a function of the water velocity. IVRs are developed to relate the AVM measured velocity to the mean channel velocity as calculated from ADCP discharge measurements. Stage is measured at each site using either an AVM acoustic transducer, a float-driven shaft encoder within a stilling well, or a pressure transducer. Bathymetric surveys relate stage to cross-sectional area of the channel. Velocity and stage data are recorded using electronic dataloggers at 5-minute intervals at each gage.

The IVRs for the Columbus Drive and Romeoville stations are given in Figures 2 and 3 below. Both equations' slopes are near 0.90 and the correlation is good. At Romeoville, the IVR is constrained to have a zero intercept. Because of the complex flow conditions present at Columbus Drive, this constraint was not imposed there.

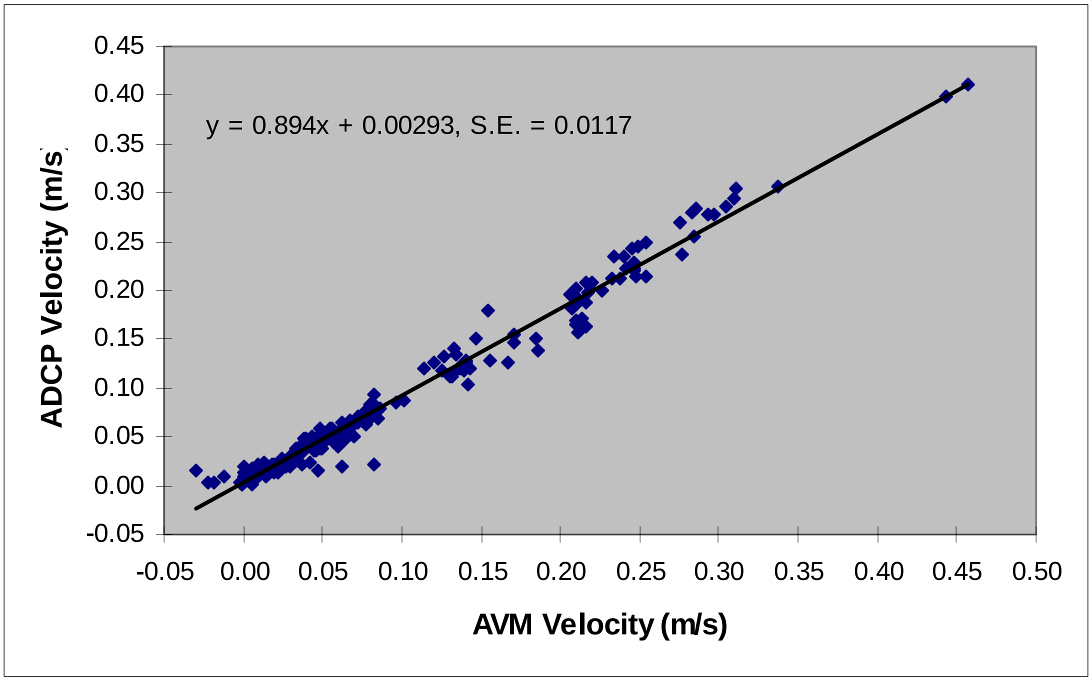

Figure 2: Index-velocity rating at CSSC at Columbus Dr.

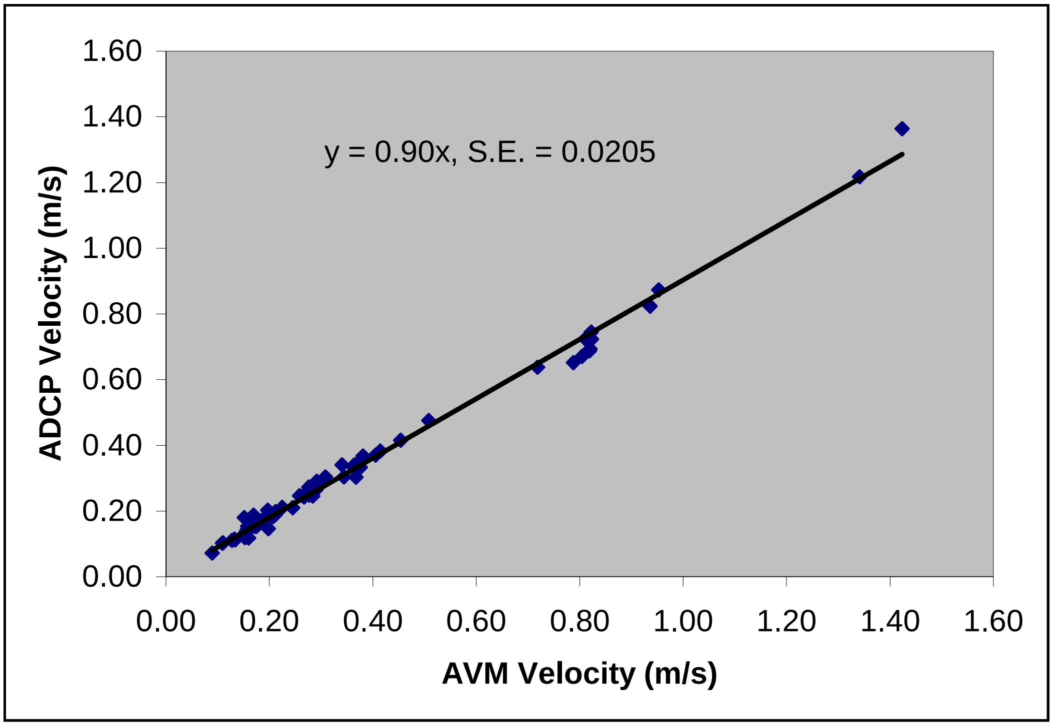

Figure 3: Index-velocity rating at CSSC at Romeoville

Parameters needed to estimate the uncertainty at these two gages using the methodology described above are given in Tables 1 and 2.

	a (m^2)	b (m)	σ_h (m)	$\bar{A}$ (m^2)	σ_A (m^2)	$\overline{V_L}$ (m/s)
Reference Equation	(6)	(6)	(7)	(9)	(11)	(10)
Columbus Dr.	470.2	63.5	0.00088	436.7	.0559	0.033
Romeoville	23.3	49.4	0.00088	405.5	.0435	0.25

Table 1: Stage, Area, and Velocity Parameters

	α (m/s)	β	σ_α (m/s)	σ_β	$\frac{\sigma_{\alpha,\beta}}{\sigma_\alpha \sigma_\beta}$	σ_ε (m/s)	σ_V at $\overline{V_L}$ (m/s)
Reference Equation	(4)	(4)	(4)	(4)	(5)	(6)	(5)
Columbus Dr.	.00293	0.894	0.00111	0.00836	-0.695	0.0117	0.0118
Romeo-ville	----	0.90	----	0.00506	----	0.0205	0.0206

Table 2: Index Velocity Relation and Regression Method Parameters

Uncertainty estimates for the two methods as a function of discharge-averaging time scale for average flow conditions are given in Table 3, and as a function of discharge at the five-minute time scale in Table 4. These results show:

1. At short time scales, the IVR standard error, σ_ε, dominates the total uncertainty if it is included in the calculations.
2. The contribution of area measurement uncertainty, as it is computed here (only arising because of stage measurement uncertainty), to the total discharge uncertainty is negligible at all time scales.
3. IVR parameter uncertainty makes up an increasing proportion of the total discharge error as the time scale increases. If σ_ε is included, it makes up a majority of the discharge uncertainty for time scales greater than about 1 day, and at all time scales if σ_ε is not included.
4. At the 5-minute time scale, discharge uncertainty varies only slightly with the value of discharge if σ_ε is included; otherwise, there is a much stronger dependence on discharge.

	Source of Uncertainty	5 min	1 hr	1 day	1 yr
Columbus Dr.	σ_ε^2	5.098	1.472	.300	.0157
	Area	.0018	.000522	.000107	.000006
	IVR Parms.	.409	.409	.409	.409
	Total – w/ σ_ε^2	5.114	1.527	.508	.410
	Total – w/o σ_ε^2	.409	.409	.409	.409
Romeoville	σ_ε^2	8.306	2.398	.489	.0256
	Area	.0099	.00286	.000583	.000030
	IVR Parms.	.519	.519	.519	.519
	Total – w/ σ_ε^2	8.322	2.453	.713	.519
	Total – w/o σ_ε^2	.519	.519	.519	.519

Table 3: Discharge Uncertainty Estimates (m^3/s) for Average Parameters as a Function of Time Scale.

	Columbus Dr.			*Romeoville*		
Q (m^3/s)	V using $\overline{A}$ (m/s)	σ_Q (m^3/s) w/ σ_ε^2	σ_Q (m^3/s) w/o σ_ε^2	V using $\overline{A}$ (m/s)	σ_Q (m^3/s) w/ σ_ε^2	σ_Q (m^3/s) w/o σ_ε^2
-50	-0.114	5.17	0.89	---	---	---
0	0.000	5.12	0.49	0.0	8.31	0.00
50	0.114	5.11	0.37	0.123	8.31	0.28
100	0.229	5.14	0.68	0.247	8.33	0.56
200	---	---	---	0.493	8.38	1.13

Table 4: Velocities and Uncertainty Estimates at the 5-Minute Time Scale for Selected Discharges

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